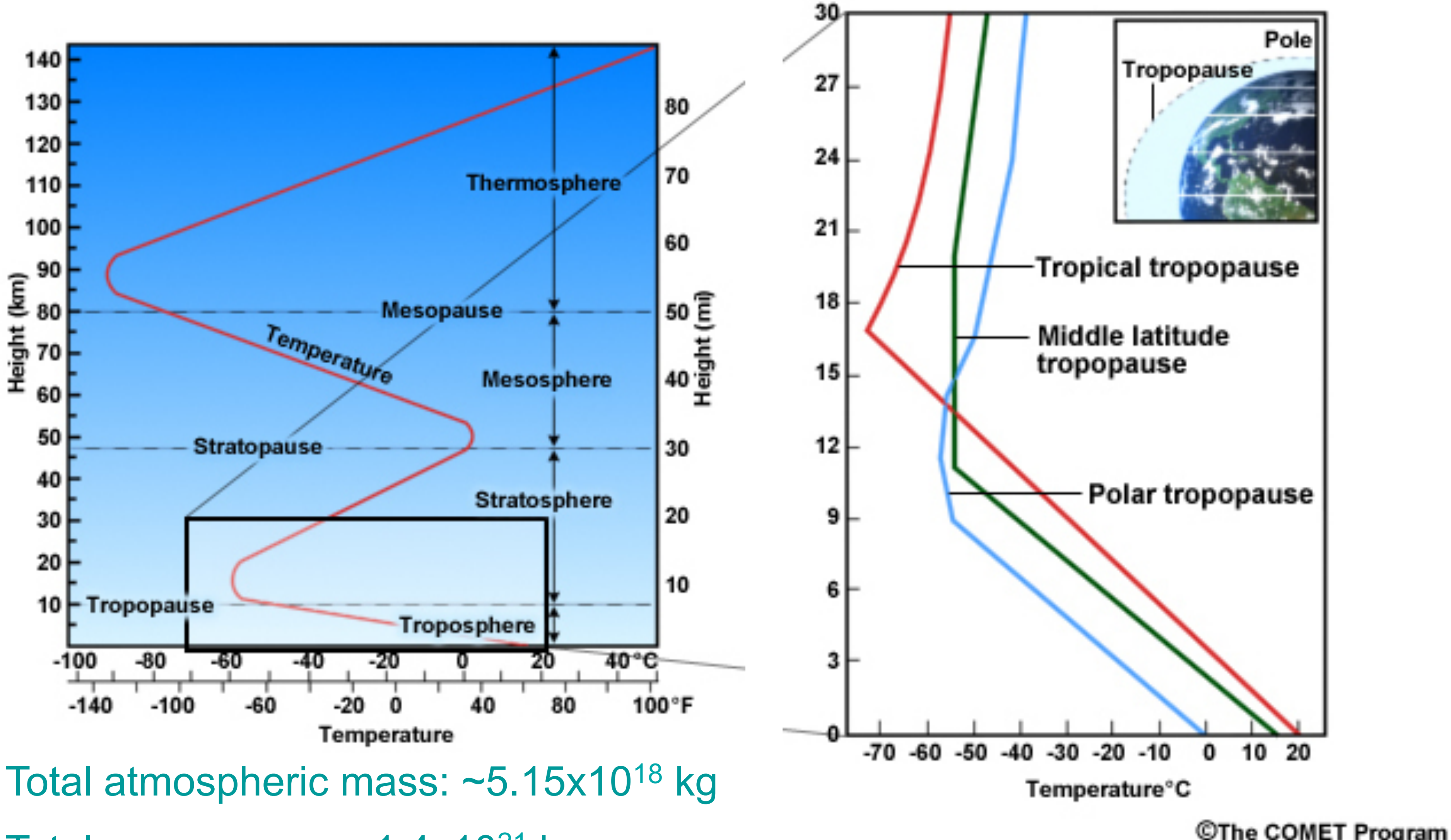
- Radiative equilibrium
- Some thermodynamics review
- Radiative-convective equilibrium

Goal: Develop a 1D description of the [tropical] atmosphere

Vertical temperature profile



Total atmospheric mass: ~$5.15x10^{18}$ kg

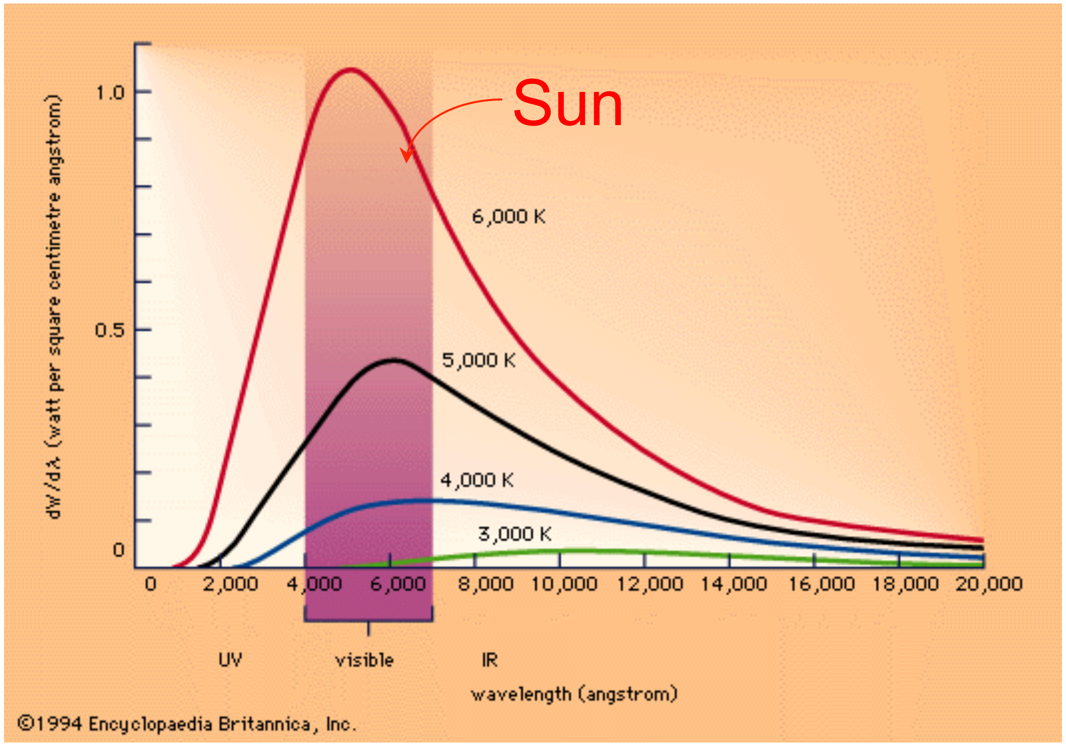
Total ocean mass: ~$1.4x10^{21}$ kg

Total earth mass: ~$5.97x10^{24}$ kg

Planck function

Radiance (R; units of J m^{-2}) as a function of wavelength (λ; units of m^{-1}) and temperature (T; units of K):

$$R(\lambda,T) = \frac{2hc^2}{\lambda^5 (e^{hc/\lambda k_B T} - 1)}$$



h [Planck's constant] = 6.625 x 10^{-34} J s^{-1}

c [speed of light] = 3.0 x 10^{8} m s^{-1}

k_B [Boltzmann's constant] = 1.38 x 10^{-23} J K^{-1}

Simple radiative balance (with no atmosphere)

- *Stefan Boltzmann equation*: $F = \sigma T^4$ *Note: Units of W m^{-2}*

  σ *= 5.67 x 10^{-8} $Wm^{-2}K^{-4}$*

- *Solar insolation*: $S_{abs} = S_0(1-\alpha_e)\pi R_e^{\ 2}$

  Note: The cross-sectional area of the Earth is relevant to how much solar energy is intercepted.

  Solar flux density: S_0 = 1370 Wm^{-2}

  Planetary albedo: $\alpha_e \approx 0.3$

  Earth radius: R_e = 6.375 x 10^6 m

- Therefore:

$$F = S_{abs}/(4\pi R_e^2) \Rightarrow T_e = [S_0(1-\alpha_e)/(4\sigma)]^{1/4}$$

Here the Earth's surface area is used.

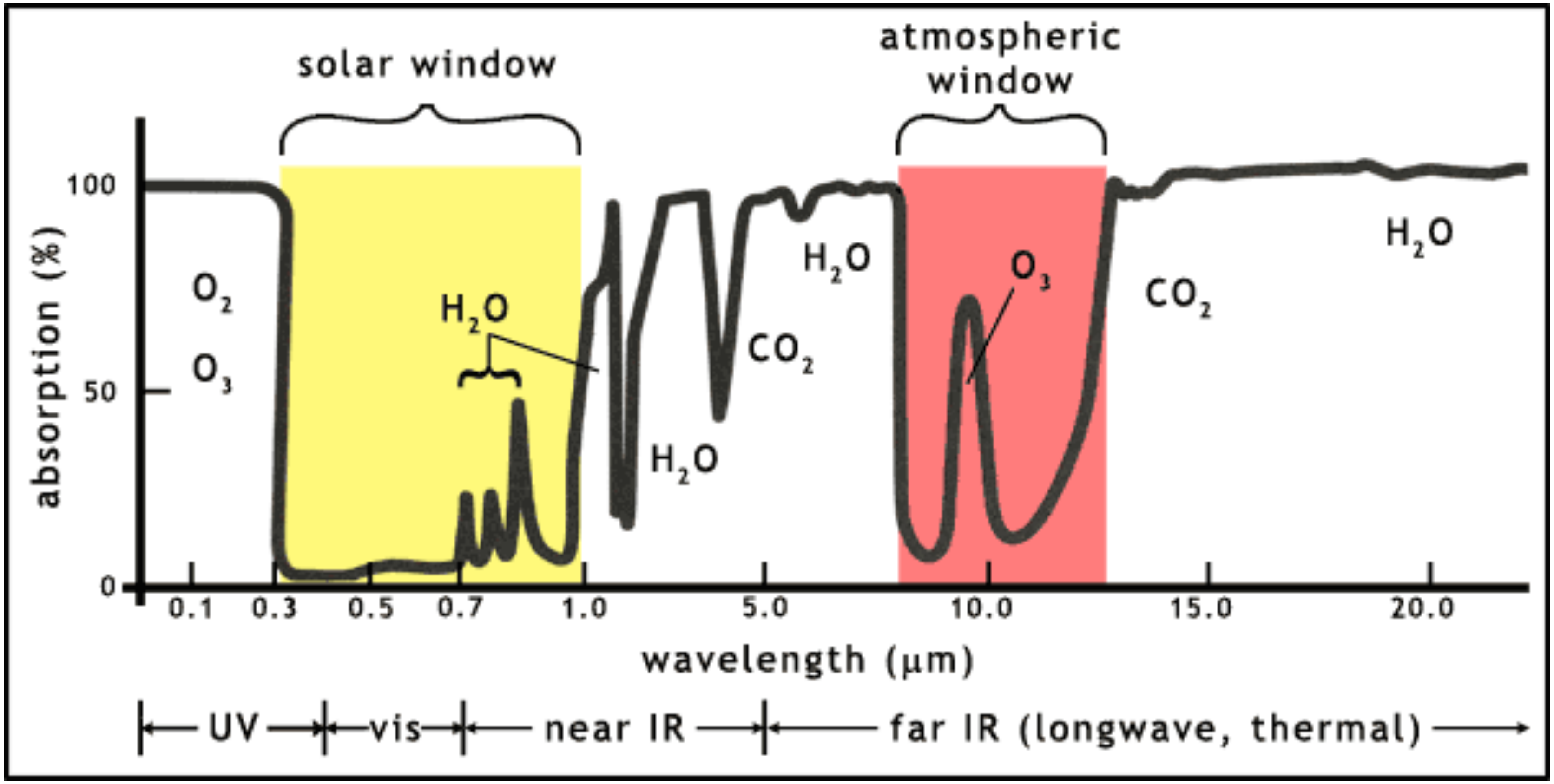
This yields an effective emission temperature of 255K (-18ºC), compared to an observed surface temperature of 288K (15ºC).

Mean tropospheric composition

Constituent	**% by volume of dry air**
N_2	78.08
O_2	20.95
CO_2	0.033 ↑
Ar	0.934
Ne	1.82×10^{-3}
He	5.24×10^{-4}
CH_4	2.0×10^{-4} ↑
Kr	1.14×10^{-4}
N_2O	5.0×10^{-5} ↑
H_2	5.0×10^{-5}
Xe	8.7×10^{-6}
O_3	4.0×10^{-6}
H_2O	0.0-4.0 [0.8]

From F.W. Taylor, *Elementary Climate Physics*

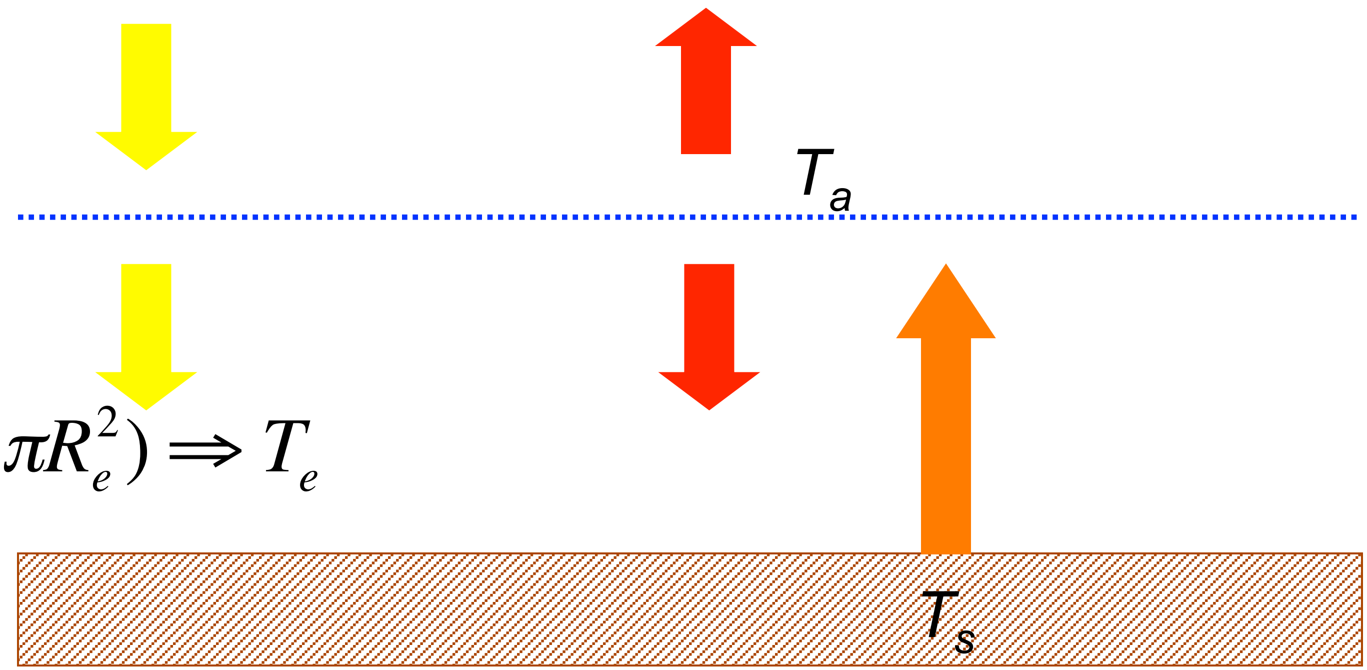
Atmospheric absorption



Turco, 2002

- In the visible, Earth's atmosphere is effectively transparent ["solar window"]
- At shorter and longer wavelengths, atmosphere is typically strongly absorbing

Simple radiative balance (opaque single layer atmosphere)



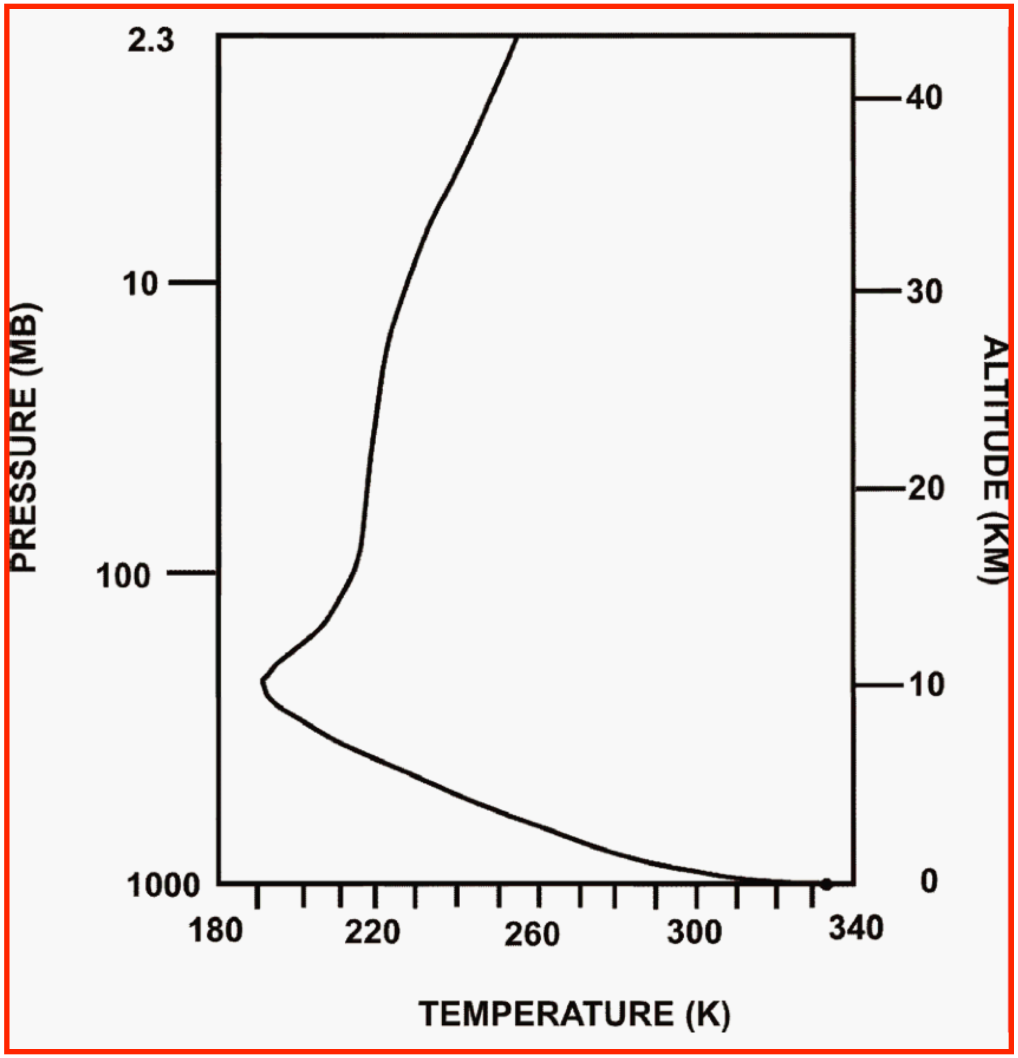
$$S_{abs}/(4\pi R_e^2) \Rightarrow T_e$$

$$T_e = T_a$$

$$\sigma T_s^4 = \sigma T_a^4 + \sigma T_e^4 \Rightarrow T_s = 2^{1/4} T_e$$

Surface temperature (~303K) is now too large…

Full 1D radiative transfer calculation



Emanuel, 2005

- Too hot (cold) close to the surface (near tropopause)
  - Thus, the lapse rate is too large
- Stratospheric temperature close to observations

We'll revisit this later…but first some thermodynamics.

Thermodynamics Review

- Entropy & thermodynamic potentials
- Hydrostatic equilibrium & buoyancy
- Stability [dry & moist adiabatic]

1st Law of Thermodynamics & Thermodynamic Identity

$$du = \delta q - \delta w$$

The notation δ denotes an inexact differential, i.e., an integral over the quantity depends on the path taken.

(1) 1st Law ⇔ energy conservation: the change in internal *du* must equal the heat added to/removed from the system *δq* and the work done on/by the system *δw*.

$$s(u,\{x_i\}) \Rightarrow \left(\frac{\partial s}{\partial u}\right)_{\{x_i\}} = \frac{1}{T}$$

(2) Definition of *T* [from specific entropy *s*, which is written as a function of *u* and a set of other variables *{x_i}*]

$$ds = \left(\frac{\partial s}{\partial u}\right)_{\{x_i\}} du + \left(\frac{\partial s}{\partial x_i}\right)_{u,\{x_{n\neq i}\}} dx_i$$

Expanding *s* as a linear function of its first partial derivatives…

$$du = Tds - T\left(\frac{\partial s}{\partial x_i}\right)_{u,\{x_{n\neq i}\}} dx_i$$

… and rearranging, using (2)…

$$du = Tds + T\left(\frac{\partial s}{d\alpha}\right)_u d\alpha = Tds - pd\alpha$$

$$= \delta q - \delta w$$

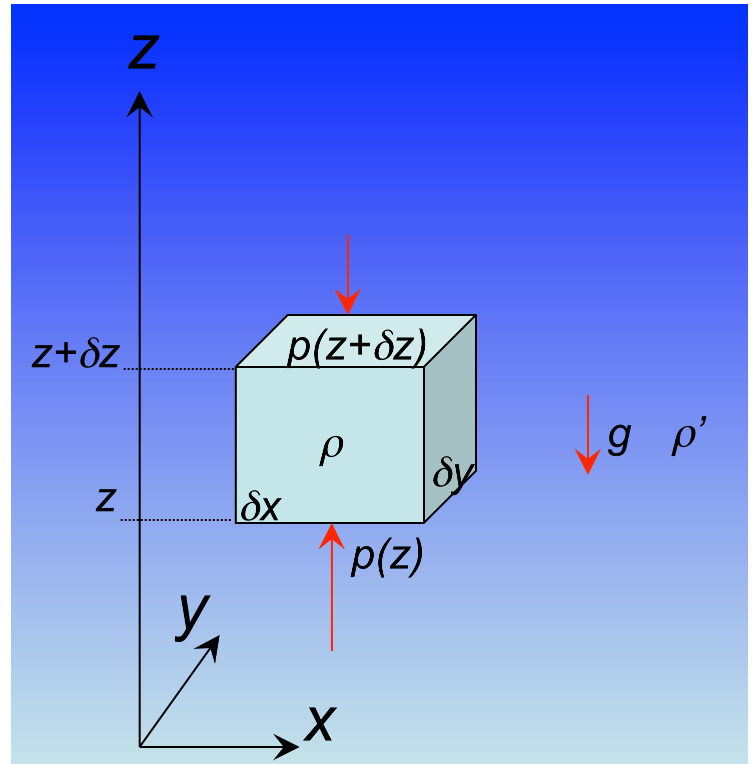
… leads to (3) the thermodynamic identity.

Here, pressure p is related to the partial derivative of s w.r.t. system specific volume.

Hydrostatic equilibrium & buoyancy

Newton's 2nd Law of Motion in the *z* direction:

$$\vec{F} = m\vec{a}$$



Consider first the case with equal environmental and parcel densities, i.e., $\rho' = \rho$

$$(\rho\delta x\delta y\delta z)\frac{d^2\delta z}{dt^2} =$$

$$-(\rho\delta x\delta y\delta z)g + p(z)\delta x\delta y - p(z+\delta z)\delta x\delta y$$

In equilibrium, and applying the limit $\delta z \rightarrow 0$,

$$0 = -\rho g - \frac{\partial p}{\partial z}$$

Consider now $\rho' \neq \rho$:

$$(\rho\delta x\delta y\delta z)\frac{d^2\delta z}{dt^2} = -(\rho\delta x\delta y\delta z)g + p(z)\delta x\delta y - p(z+\delta z)\delta x\delta y$$

$$\Rightarrow \frac{d^2\delta z}{dt^2} = -g - \frac{1}{\rho}\frac{\partial p}{\partial z} = g\left(\frac{\rho' - \rho}{\rho}\right) \equiv B$$ Buoyancy

For a dry adiabatic displacement

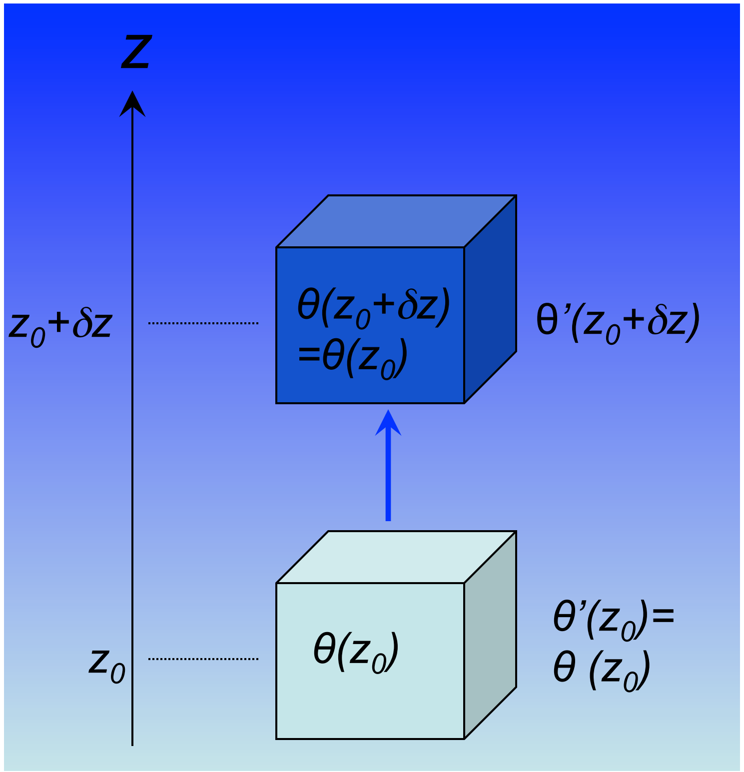
Consider a parcel undergoing a dry adiabatic displacement from an initial position to a new position separated by a distance δz. At the new position:

$$\frac{dw}{dt} = g\frac{\rho' - \rho}{\rho} = g\frac{T - T'}{T'} = g\frac{\theta - \theta'}{\theta'}$$

$$= g\left(\frac{\theta}{\theta'} - 1\right) = g\left(\frac{\theta}{\theta + \delta z \frac{\partial\theta}{\partial z}} - 1\right)$$

$$\approx g\left[\left(1 - \frac{1}{\theta}\frac{\partial\theta}{\partial z}\delta z\right) - 1\right] = -\frac{g}{\theta}\frac{\partial\theta}{\partial z}\delta z$$

$$\Rightarrow \frac{d^2\delta z}{dt^2} + N^2\delta z \approx 0; \quad N^2 = \frac{g}{\theta}\frac{\partial\theta}{\partial z}\delta z$$



N is the Brunt-Väisäila frequency.

Dry adiabatic displacement in terms of lapse rates

$$\frac{dw}{dt} = g\frac{\rho' - \rho}{\rho} = g\frac{T - T'}{T'}$$

$$\approx -g\frac{(\Gamma_d - \Gamma)}{T'}\delta z$$

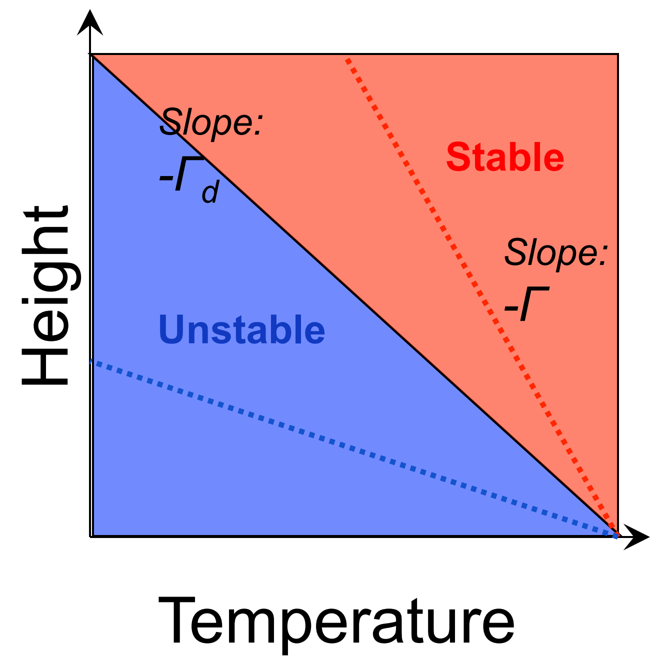
$$T(z_0 + \delta z) \approx T_e(z_0) - \Gamma_d \delta z$$

$$T'(z_0 + \delta z) \approx T(z_0) - \Gamma \delta z$$

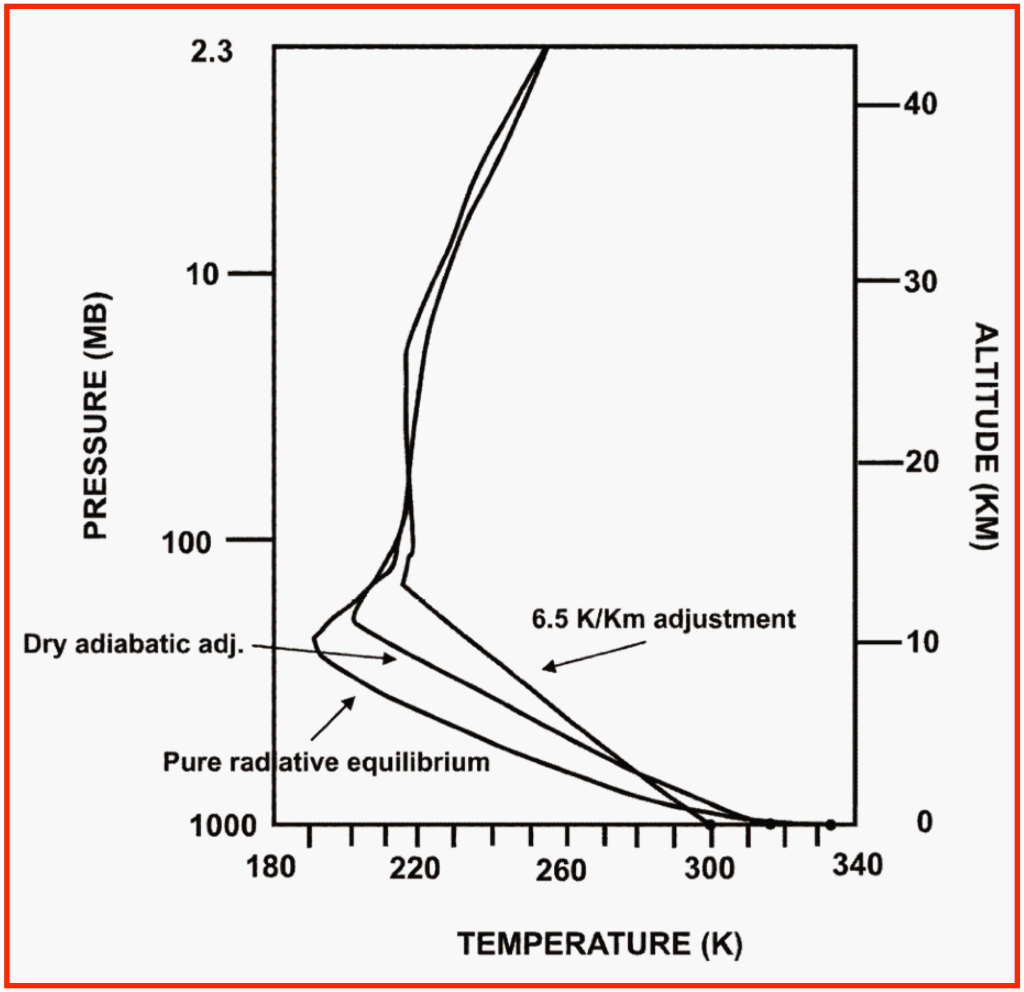
$$\frac{d^2\delta z}{dt^2} + N^2(z)\delta z = 0 \quad N(z) = \sqrt{g\frac{\Gamma_d - \Gamma}{T'(z)}}$$

If $\Gamma < \Gamma_d$, $N^2(z) > 0$, so solutions to the perturbation equation are oscillatory (sinusoids). In this case, the equilibrium is **stable**.

If $\Gamma > \Gamma_d$, $N^2(z) < 0$, so solutions to the perturbation equation are exponential (hyperbolic sine/cosine). In this case, the equilibrium is **unstable**.



Revisiting radiative equilibrium…



Emanuel, 2005

- Pure radiative equilibrium is in fact unstable for conditions in the troposphere [but is reasonable in the stratosphere].
  - Need to account for vertical heat transport via convection [more shortly and later]
- Γ_d significantly exceeds the observed lapse rate in the atmopshere (~6.5K/km)
  - Modification of the lapse rate by moisture

Moist variables

IGL for dry air (denoted $_d$) and water vapor (denoted $_v$):

$$p_d \alpha = R_d T$$
$$e \alpha = R_v T$$

Recall:

$$\left[\frac{R_d}{R_v} = \frac{m_v}{m_d} \approx \frac{18}{28.9} = 0.622 \right]$$

By Dalton's Law of Partial Pressures, the total pressure of moist air [mixture of dry air+vapor] is:

$$p = p_d + e$$

The density of moist air is:

$$\rho = \rho_d + \rho_v = \frac{p}{R_d T}\left(1 - 0.378\frac{e}{p}\right)$$

Virtual temperature (T_v): temperature to which dry air must be raised to have the same density as moist air at the same pressure

$$T_v = T\left[1 - 0.378\frac{e}{p}\right]^{-1}$$

Specific humidity (q):

$$q = \frac{\rho_v}{\rho} = \frac{0.622e}{p - 0.378e} \approx 0.622\frac{e}{p}$$

Using the definitions of q and T_v, the equation of state of moist air is approximately:

$$p \approx \rho R_d T(1 + 0.61q)$$

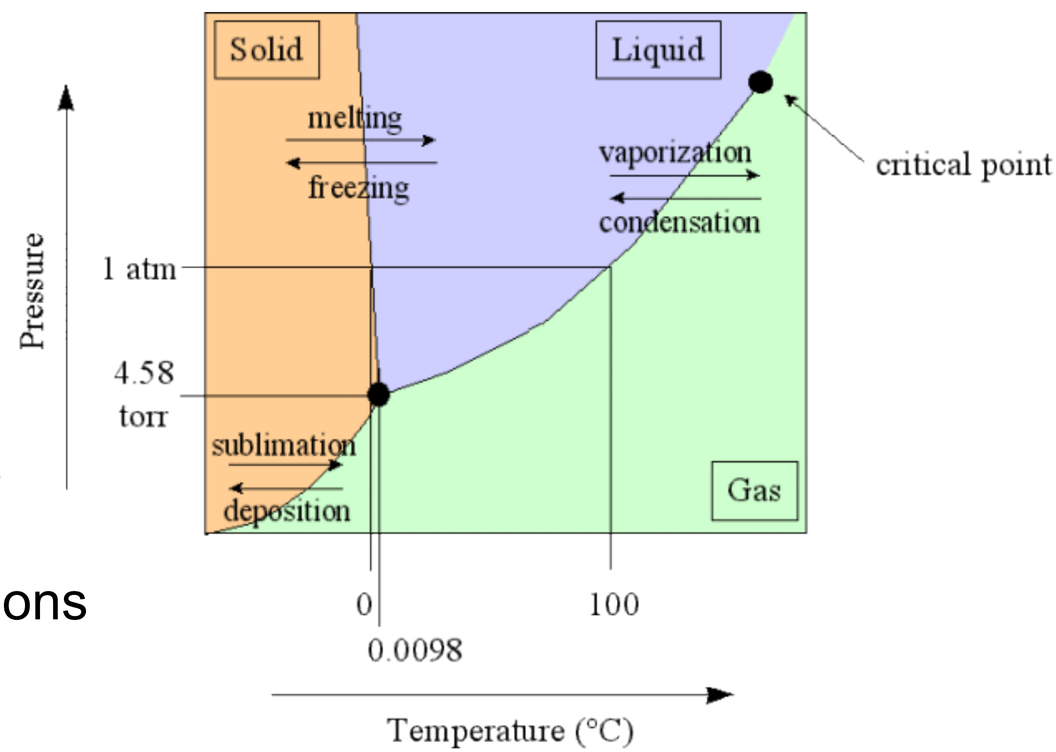
Saturation vapor pressure

Consider the relative humidity (*rh*): $rh = \frac{e}{e_s}$

Here, the saturation vapor pressure (e_s) is the vapor pressure at which condensation occurs; recall that e_s is a monotonic function of temperature.

The *Clausius-Clapeyron* (CC) equation governs the temperature dependence of e_s for two-phase equilibrium (the gas-liquid coexistence line on the phase diagram):

$$\frac{de_s}{T} = \frac{L}{T(\alpha_2 - \alpha_2)}$$



If 1 denotes the condensed phase (solid or liquid) and 2 the gaseous phase, $\alpha_2 >> \alpha_1$. From prior definitions and after integration:

$$e_s(T) \propto \exp\left[-0.622 \frac{L}{R_d T}\right]$$

Adiabatic processes in a moist atmosphere

Recall that θ is the temperature a parcel of air would have if displaced, adiabatically and reversibly, to a reference pressure p_0 [typically, 1000 mb]:

$$\theta = T\left(\frac{p_0}{p}\right)^{\kappa} ; \kappa = \frac{R_d}{c_p}$$

[To derive: apply 1st law for an adiabatic process (ds=0), use definition of internal energy and IGL, and integrate.]

The (dry) entropy can be expressed in terms of θ:

$$\Delta s_d = c_p \ln\theta \qquad d\Delta s_d = 0 \Leftrightarrow d\theta = 0$$

The Δ notation indicates that specific entropy is defined up to an arbitrary constant.

Now, for an adiabatic upward displacement of *saturated* air, condensation will occur, leading to a release of latent heat in the amount $-Ldq_s$. The entropy change associated with this release is $d\Delta s_c=(-Ldq_s)/T$. Equating $d\Delta s_d$ and $d\Delta s_c$ and integrating gives the equivalent potential temperature θ_e:

$$c_p d\ln\theta = -\frac{L_c dq_s}{T} \approx -d\left(\frac{L_c q_s}{T}\right) \qquad \theta_e = \theta \exp\left[\frac{Lq_s}{c_p T}\right]$$

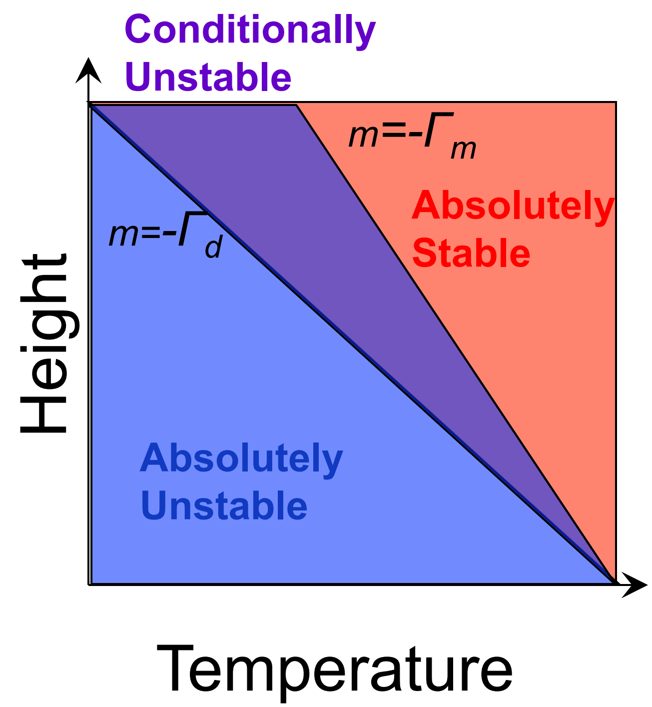
Moist Stability

Starting with the vertical acceleration and using the definition of potential temperature, it can be shown that:

$$\frac{d^2\delta z}{dt^2} + N^2(z)\delta z = 0 \Rightarrow \frac{d^2\delta z}{dt^2} + \left(\frac{g}{\theta}\frac{\partial\theta}{\partial z}\right)\delta z = 0$$

Thus: $\frac{\partial\theta}{\partial z} > 0$ *Stable*

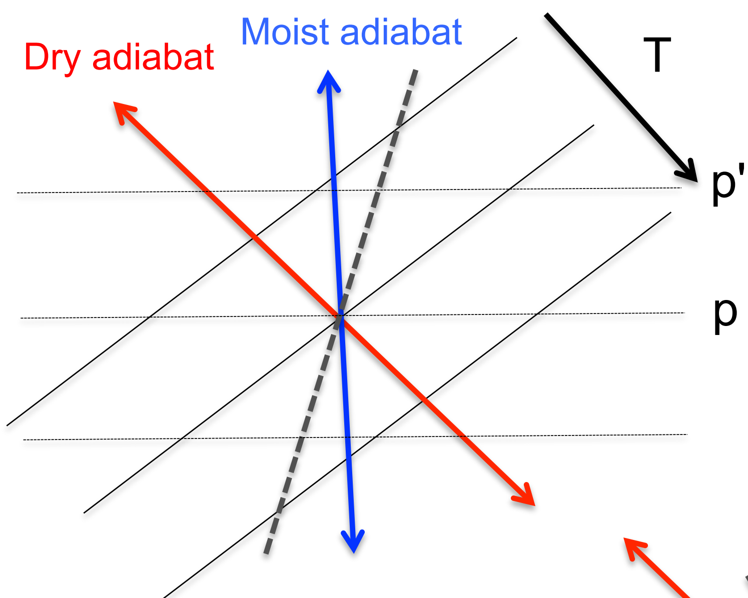
$\frac{\partial\theta}{\partial z} < 0$ *Unstable*



Equivalent potential temperature can be used to evaluate stability for a moist atmosphere; by analogy, a moist adiabatic lapse Γ_m rate can be defined. A region of conditional instability emerges, for $\Gamma_m < \Gamma < \Gamma_d$. For unsaturated air, this region is stable; for saturated air, it is unstable.

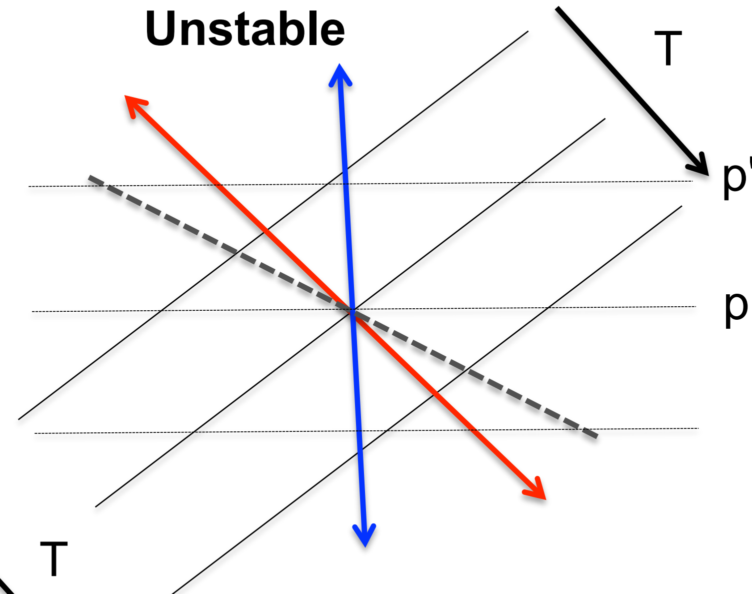
Stability in a moist atmosphere

Stable

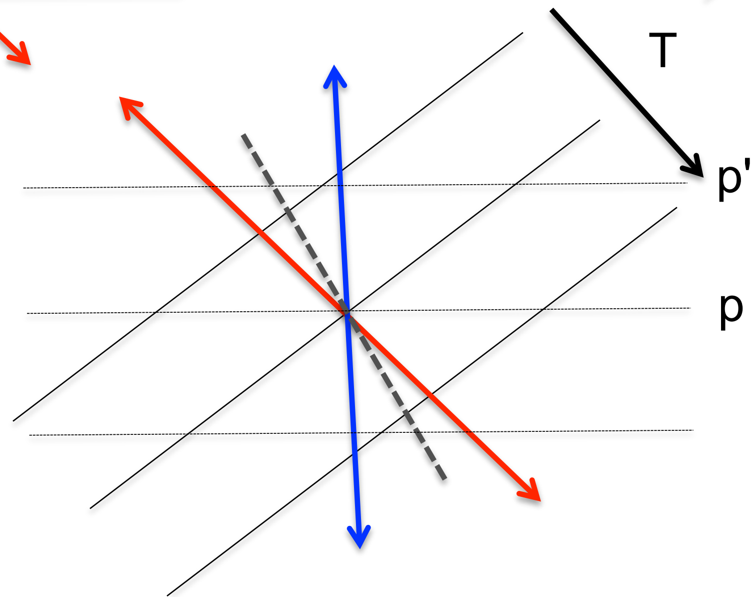


Environment at p’ warmer than parcel undergoing either dry or moist adiabatic ascent from p

Unstable



Environment at p’ cooler than parcel undergoing either dry or moist adiabatic ascent from p



Conditionally Unstable

Environment at p’ warmer than parcel undergoing dry adiabatic ascent but cooler than for moist adiabatic ascent

Overview of convection [more later]

- Convection is an important source of atmospheric heating associated with H_2O phase change. As water vapor condenses, it releases latent heat.
- The vertical motion associated with convection is an important control on the space-time behavior of H_2O vapor, other tracers, aerosols, and clouds.
- *Global* hydrologic cycle balance implies balance between *global* mean precipitation and evaporation rates [~3 mm day^{-1}]…although significant regional [sub-global] heterogeneity exists.
- Evaporation is an important component of the surface energy budget.

Convective adjustment

- Eliminates atmospheric column instability
- Environmental conditions that favor instability:
  - **Cooling** aloft via cold air advection or longwave radiative cooling
  - **Warming** of surface via warm air advection or solar heating
  - *Lifting* of air mass through low-level convergence